All work must be shown to be awarded full credit.
Provide exact solutions to all problems, unless otherwise stated. A scientific calculator is allowed.

Student Name: KEY **ID:**
Instructor: **Mundy-Castle**

Exam Score: ________

1) Find the following limits analytically.

a) $\lim_{x\to\frac{\pi}{4}} \frac{1-\tan x}{\sin x - \cos x} = \lim_{x\to\frac{\pi}{4}} \frac{\left(1-\frac{\sin x}{\cos x}\right)\cos x}{(\sin x - \cos x)\cos x}$

$= \lim_{x\to\frac{\pi}{4}} \frac{\cos x - \sin x}{(\sin x - \cos x)\cos x} = \lim_{x\to\frac{\pi}{4}} \frac{-(\sin x - \cos x)}{(\sin x - \cos x)\cos x}$

$= \lim_{x\to\frac{\pi}{4}} \frac{-1}{\cos x} = \frac{-1}{\sqrt{2}/2} = -\frac{2}{\sqrt{2}} = -\sqrt{2}$

b) $\lim_{x\to 0} \frac{\frac{1}{x+4}-\frac{1}{4}}{x}$

$\lim_{x\to 0} \frac{\frac{4}{4}\frac{1}{(x+4)} - \frac{1}{4}\frac{(x+4)}{(x+4)}}{x/1} = \lim_{x\to 0} \frac{4-(x+4)}{4(x+4)} \cdot \frac{1}{x}$

$= \lim_{x\to 0} \frac{-x}{4(x+4)} \cdot \frac{1}{x} = \lim_{x\to 0} \frac{-1}{4(x+4)} = \frac{-1}{4(4)} = -\frac{1}{16}$

All work must be shown to be awarded full credit.
Provide exact solutions to all problems, unless otherwise stated. A scientific calculator is allowed.

2) Find the constants a and b such that $f(x) = \begin{cases} 2a\ , & x \le -1 \\ x-1, & -1 < x < 3 \\ -2b\ , & x \ge 3 \end{cases}$ is continuous on the entire real line.

Need $\lim_{x\to -1} f(x)$ to exist, so we need $\lim_{x\to -1^-} f(x) = \lim_{x\to -1^+} f(x)$

$\lim_{x\to -1^-} f(x) = \lim_{x\to -1^-} (2a) = 2a$

so $2a = -2$, $a = -1$

$\lim_{x\to -1^+} f(x) = \lim_{x\to -1^+} (x-1) = -1-1 = -2$

Also need $\lim_{x\to 3} f(x)$ to exist, so we need $\lim_{x\to 3^-} f(x) = \lim_{x\to 3^+} f(x)$

$\lim_{x\to 3^-} f(x) = \lim_{x\to 3^-} (x-1) = 3-1 = 2$

so $-2b = 2$, $b = -1$

$\lim_{x\to 3^+} f(x) = \lim_{x\to 3^+} (-2b) = -2b$

All work must be shown to be awarded full credit.
Provide exact solutions to all problems, unless otherwise stated. A scientific calculator is allowed.

3) Find the first derivative of the following functions.

a) $f(x) = (\sin x)\sqrt{x^4 + 2x - 1}$
$= \sin x\,(x^4+2x-1)^{1/2}$

Product Rule

$$f'(x) = (\cos x)\sqrt{x^4+2x-1} + \sin x\left[\frac{1}{2}(x^4+2x-1)^{-1/2}(4x^3+2)\right]$$

$$= (\cos x)\sqrt{x^4+2x-1} + \frac{(\sin x)(2x^3+1)}{\sqrt{x^4+2x-1}}$$

b) $g(x) = \tan^2\left(\frac{3x}{2x-1}\right) = \left[\tan\left(\frac{3x}{2x-1}\right)\right]^2$

$$g'(x) = 2\tan\left(\frac{3x}{2x-1}\right)\sec^2\left(\frac{3x}{2x-1}\right)\left[\frac{3(2x-1)-(3x)(2)}{(2x-1)^2}\right]$$

$$= \left[\frac{-6}{(2x-1)^2}\right]\tan\left(\frac{3x}{2x-1}\right)\sec^2\left(\frac{3x}{2x-1}\right)$$

All work must be shown to be awarded full credit.
Provide exact solutions to all problems, unless otherwise stated. A scientific calculator is allowed.

4) Use implicit differentiation to find dy/dx for $3x^2y + 2xy = 3y^2$.

$$\frac{d}{dx}\left[3x^2y + 2xy\right] = \frac{d}{dx}\left[3y^2\right]$$

$$6xy + 3x^2y' + 2y + 2xy' = 6yy'$$

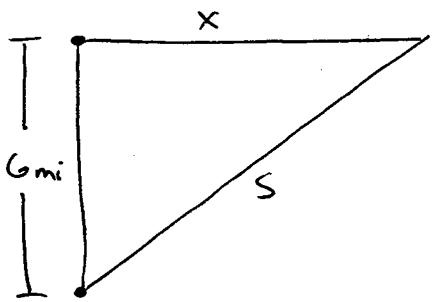
$$3x^2y' + 2xy' - 6yy' = -6xy - 2y$$

$$y'\left(3x^2 + 2x - 6y\right) = -6xy - 2y$$

$$y' = \frac{-6xy - 2y}{3x^2 + 2x - 6y}$$

All work must be shown to be awarded full credit.
Provide exact solutions to all problems, unless otherwise stated. A scientific calculator is allowed.

5) An airplane is flying on a flight path that will take it directly over a radar tracking station. The plane is 6 vertical miles above the station, x horizontal miles from the station, and s miles on a direct path to the station. If s is decreasing at a rate of 400 miles per hour when $s = 10$ miles, what is the speed of the airplane?



Given: $\frac{ds}{dt} = -400$ mph

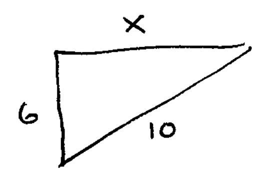
Want: $\frac{dx}{dt}$ when $s = 10$ mi.

Relation: $x^2 + 6^2 = s^2$

differentiate: $2x\frac{dx}{dt} = 2s\frac{ds}{dt}$

$\frac{dx}{dt} = \frac{s}{x}\frac{ds}{dt}$

Find x:



$x^2 + 6^2 = 10^2$
$x^2 = 64, \; x = 8$

$\left.\frac{dx}{dt}\right|_{s=10} = \frac{10}{8}(-400)$

$= -500$

speed of plane is 500 mph